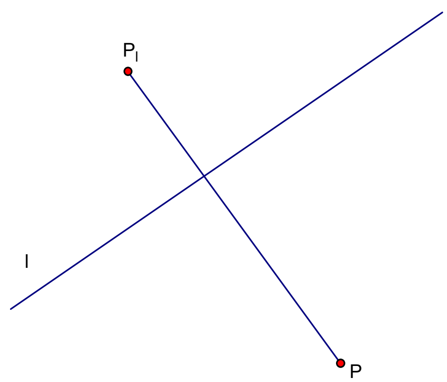
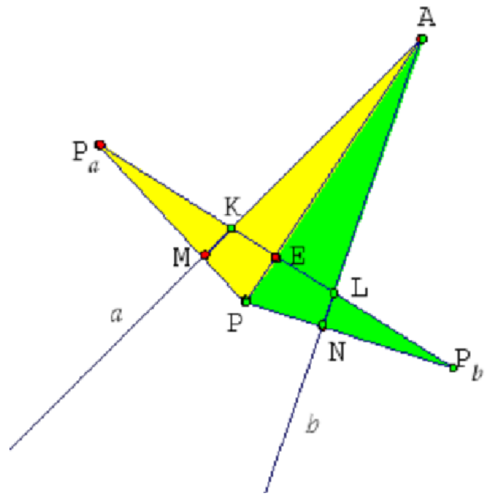
**Inverse to Erdos-Mordell inequality**.
https://www.linkedin.com/feed/update/urn:li:activity:7157094333530177536?utm_source=share&utm_medium=member_desktop
Let $a, b, c$ denote the lengths of the sides of a triangle $ABC$, let $d_a,\ d_b, d_c$ denote the distances from an arbitrary point $P$ inside the triangle to sides $BC, CA, AB$ , respectively, and let $R_a := PA, R_b := PB, R_c := PC$. Prove that:

$$\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} \leq \frac{1}{2}\left(\frac{1}{d_a} + \frac{1}{d_b} + \frac{1}{d_c}\right).$$

**Solution by Arkady Alt**, **San Jose**, **California**, **USA**.
**Definition**.
For any line $l$ on the plane and any point $P \notin l$ we denote via $P_l$ such point laying in the half-plane distinct from half-plane marked by point $P$ that $PP_l \perp l$ and $PP_l = \dfrac{1}{dist(P, l)}$.
This point $P_l$ we will call "Involution of $P$ with respect to $l$" (**Pic**.**1**)




**Pic**.**1**
Let $P$ be the interior point in the angle $\angle A$ defined by the two half-lines $a$ and $b$.
Let $d_a$ and $d_b$ be distances prom point $P$ to lines $a$ and $b$ respectively and
$R_A$ be distance between $P$ and $A$, i.e. $d_a = PM, d_b = PN, R_A = PA$. (**Pic**.**2**)
We will prove
**Lemma**.
Let $P_a$ and $P_b$ be involutions of $P$ with respect to $a$ and $b$ respectively.
Then $P_aP_b \perp PA$ and $PE = \dfrac{1}{R_A}$ where $E$ is intersection point of $P_aP_b$ and $PA$.
**Proof**.




**Pic.2**

Let $P_aE_1$ and $P_bE_2$ be perpendiculars from $P_a$ and $P_b$ to $\overleftrightarrow{PA}$ respectively ($E_1, E_2 \in \overleftrightarrow{PA}$ ). Since $\angle PP_aE_1 = \angle PMA$ and $\angle PP_bE_2 = \angle PNA$ (as the angles which constructed by mutually perpendicular sides) then $\triangle PP_aE_1$similar to $\triangle PMA$ and $\triangle PP_bE_2$ similar to $\triangle PNA$ and from similarity follows

$$\frac{PE_1}{PP_a} = \frac{PM}{PA} \Leftrightarrow \frac{PE_1}{\frac{1}{d_a}} = \frac{d_a}{R_A} \Leftrightarrow PE_1 = \frac{1}{R_A} \text{ and}$$

$$\frac{PE_2}{PP_b} = \frac{PN}{PA} \Leftrightarrow \frac{PE_2}{\frac{1}{d_b}} = \frac{d_b}{R_A} \Leftrightarrow PE_2 = \frac{1}{R_A}.$$

Hence, $PE_1 = PE_2$ and $E := E_1 = E_2$ is intersection point of $P_aP_b$ with $PA$ and $PE = \frac{1}{R_A}$.

Let $A_1, B_1, C_1$ be involution points for $P \in \triangle ABC$ with respect to lines $\overleftrightarrow{BC}, \overleftrightarrow{CA}, \overleftrightarrow{AB}$ respectively. Let $R_a' = PA_1 = \frac{1}{d_a}, R_b' = PB_1 = \frac{1}{d_b}, R_c' = PC_1 = \frac{1}{d_c}$ and $d_a', d_b', d_c'$ be distances from $P$ to sides $B_1C_1, C_1A_1, A_1B_1$.

Applying lemma we obtain $d_a' = \frac{1}{R_a}, d_b' = \frac{1}{R_b}, d_c' = \frac{1}{R_c}$ and by replacing $(R_a, R_b, R_c, d_a, d_b, d_c)$ in **Erdös-Mordell Inequality**

$$R_a + R_b + R_c \ \geq 2(d_a + d_b + d_c)$$

with $(R_a', R_b', R_c', d_a', d_b', d_c') = \left(\frac{1}{d_a}, \frac{1}{d_b}, \frac{1}{d_c}, \frac{1}{R_a}, \frac{1}{R_b}, \frac{1}{R_c}\right)$ we obtain

$$\sum_{cyc} R_a' \geq 2\sum_{cyc} d_a' \Leftrightarrow \sum_{cyc} \frac{1}{d_a} \geq 2\sum_{cyc} \frac{1}{R_a}.$$